Time, Frequency, Energy, and Bits

Hutter RNN Project

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1 Setup: Carrier Frequency

Assume a carrier frequency f_p that defines a fundamental period:

$$T_p = \frac{1}{f_p}$$

For concreteness, let $f_p = 1$ Hz, so $T_p = 1$ second.

Within one period, phase $\phi \in [0, 2\pi)$ maps to the unit interval:

$$t = \frac{\phi}{2\pi} \in [0, 1)$$

Everything in the system operates on some frequency. The spectrum decomposes signals into frequency components.

2 Fundamental Relations

2.1 Energy and Frequency

Planck's relation:

$$E = hf$$

where h is Planck's constant. Energy is proportional to frequency.

2.2 Frequency and Time

Frequency is the reciprocal of period:

$$f = \frac{1}{T}$$

2.3 Energy and Time

Combining:

$$E = \frac{h}{T}$$

Energy and time are reciprocals (up to h).

The energy-time uncertainty principle:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

2.4 Power

Power is energy per unit time:

$$P = \frac{E}{T} = E \cdot f = hf^2$$

Integrating power over the period:

$$\int_0^{T_p} P\,dt = E_{\text{total}}$$

3 Information and Energy

3.1 Landauer's Principle

Erasing one bit of information requires minimum energy:

$$E_{\text{bit}} = k_B T \ln 2$$

where k_B is Boltzmann's constant and T is temperature.

At fixed temperature, **bits are proportional to energy**:

$$\text{bits} = \frac{E}{k_B T \ln 2}$$

3.2 Entropy Rate

If events arrive at rate f (events per second), and each event carries H bits of entropy:

$$\dot{H} = H \cdot f \quad \text{[bits/second]}$$

Integrating over the period:

$$\int_0^{T_p} \dot{H}\,dt = H \cdot f \cdot T_p = H \cdot \frac{f}{f_p} \quad \text{[bits per period]}$$

4 The Depth Limit Reinterpreted

4.1 Original Form

The depth limit from float32 precision:

$$d_{\max} = \frac{24}{-\log_2 p_{\text{avg}}} = \frac{24}{H_{\text{avg}}}$$

This says: we can process at most $d_{\max}$ events before precision runs out.

4.2 Energy Interpretation

Rewrite as:

$$d_{\max} \cdot H_{\text{avg}} = 24 \text{ bits}$$

The left side is total entropy processed. The right side is the "bit budget."

Via Landauer:

$$E_{\text{budget}} = 24 \cdot k_B T \ln 2$$

The depth limit is an energy budget.

Each event "spends" $H_{\text{avg}} \cdot k_B T \ln 2$ energy. After $d_{\max}$ events, the budget is exhausted.

4.3 Time Interpretation

If events arrive at rate f:

$$T_{\max} = \frac{d_{\max}}{f} = \frac{24}{H_{\text{avg}} \cdot f}$$

This is the maximum temporal horizon—how far back we can "remember."
Higher entropy per event (H_{avg}) or higher event rate (f) reduces the horizon.

5 Unit Analysis Summary

Quantity	Symbol	Units
Frequency	f	[time^{-1}]
Period	$T = 1/f$	[time]
Energy	$E = hf$	[energy]
Power	$P = E/T = hf^2$	[energy $\cdot$ time^{-1}]
Entropy	H	[bits]
Entropy rate	$\dot{H} = Hf$	[bits $\cdot$ time^{-1}]
Bit-energy	$E_{\text{bit}} = k_B T \ln 2$	[energy]
Bits from energy	E/E_{bit}	[bits]

6 The Chain of Proportionalities

$$\text{bits} \propto \text{energy} \qquad \text{(Landauer)} \qquad (1)$$

$$\text{energy} = h \times \text{frequency} \qquad \text{(Planck)} \qquad (2)$$

$$\text{frequency} = 1/\text{time} \qquad \text{(definition)} \qquad (3)$$

Therefore:

$$\boxed{\text{bits} \propto \frac{h}{\text{time}}}$$

More time (lower frequency) means fewer bits per unit action. More bits requires more energy requires higher frequency requires finer time resolution.

7 Implications

7.1 For RNNs

The hidden state h_t carries information through time. Each timestep "costs" entropy. The precision limit (24 bits for float32) sets the total budget.

W_{hh} injection fails because temporal patterns require carrying bits across multiple steps, exceeding the budget.

7.2 For Arithmetic Coding

The interval $[L, H) \subset [0, 1)$ narrows with each symbol. The quotient $Q = 1/(H - L)$ grows. When $Q > 2^{24}$, precision is exhausted.

7.3 For the Brain

If neural computation operates on a carrier frequency f_p, the energy budget per period constrains how much can be computed.

Higher-frequency oscillations (gamma, 40 Hz) enable more computation per second but cost more energy.

Lower-frequency oscillations (theta, 6 Hz) are more efficient but slower.

8 Connection to Zeta

The Riemann zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_{p} \frac{1}{1 - p^{-s}}$$

The sum over integers equals the product over primes—additive structure equals multiplicative structure.

In our framework:

- Integers n = events encoded as natural numbers (E $\to$ N)
- Primes p = irreducible factors (atomic event spaces)
- The duality: time-domain (sum) $\leftrightarrow$ frequency-domain (product)

The π in $\zeta(2) = \pi^2/6$ connects to:

- The unit circle (phase $\in [0, 2\pi)$)
- Fourier transform (integration over $e^{2\pi i f t}$)
- Gaussian distributions (which arise from maximum entropy)

9 Summary

Time, frequency, energy, and bits are all aspects of the same thing. The depth limit $d_{\max} = 24/H$ is:

- A bit budget (information)
- An energy budget (thermodynamics)
- A temporal horizon (time)
- A precision limit (computation)

Fourier duality (time $\leftrightarrow$ frequency) underlies the Bayes/Thermo duality (probability $\leftrightarrow$ microstates).