The Factor Map and Weight Construction Scale: R^2=0.83 Is an Architectural Invariant

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Abstract

We apply the factor map analysis and weight construction experiments from the 1024-byte model to the enwik9 model (110M checkpoint). The central result: the 2-offset conjunction R^2 is 0.830, nearly identical to the 1024-byte value of 0.837. This is not because the models are similar—they are architectural opposites (deep/concentrated vs. shallow/distributed)—but because the factor map structure is a property of the 128-hidden tanh RNN architecture, not of the training data. Three further findings: (1) word-length subtraction is 13× less catastrophic (+0.54 vs +7.3 bpc), confirming the shallow/distributed character; (2) the Hebbian correlation splits: overall r drops (0.38 vs 0.56), but large-weight r increases (0.77 vs 0.74); (3) the fully analytic construction achieves 4.21 bpc with zero optimization, closing the loop for a second model.

1 Setup

We use the same analysis tools from the 20260208–20260209 archives on the enwik9 checkpoint `epoch1_110M.bin` (110M bytes, Adam optimizer, gradient clipping, 2.81 bpc eval on rolling average). Evaluation data: the first 1024 bytes of enwik9 (the model starts from $h = 0$ during both training and evaluation, so these positions are directly comparable). The model achieves 6.42 bpc on these bytes, much worse than the 1024-byte sat-rnn (0.079 bpc), because the enwik9 model has processed 110M bytes of data since seeing them.

2 P4: Factor Map R^2

Finding 1 (P4 is WRONG—R^2 does not decrease)**.** *The prediction that 2-offset conjunction R^2 would drop to 0.4–0.6 is spectacularly wrong. The enwik9 model has R^2=0.830, within 1% of the 1024-byte model's 0.837.*

	1024B	**enwik9**
Mean 2-offset R^2	0.837	**0.830**
Neurons $R^2 \geq 0.90$	many	7
Neurons $R^2 \geq 0.80$	120/128	98/128
Neurons $R^2 \geq 0.70$	—	128/128
Mean single-offset R^2	—	0.416
BPC gain captured	87%	100.2%

Table 1: Factor map R^2. The 2-offset conjunction structure is preserved across models. The enwik9 model captures 100% of the BPC gain via conditional means, matching the actual RNN output.

Offset pair distribution. The dominant offset pairs shift but remain structurally similar:

Pair	1024B	enwik9
(1,7)	52/128	30/128
(1,8)	—	45/128
(1,3)	—	27/128
(1,12)	—	8/128
(1,20)	—	6/128
other	76/128	12/128

Offset 1 is universal: every neuron in the enwik9 model uses offset 1 as one of its two conjunction offsets. The 1024-byte model had a dominant (1,7) pairing; the enwik9 model distributes more evenly across (1,8), (1,7), and (1,3).

Interpretation. Each neuron computes $h_j \approx E[h_j \mid \text{data}[t-d_1], \text{data}[t-d_2]]$, a conjunction of two past byte values. This structure is architectural: the 128-hidden tanh RNN, regardless of training data or regime, learns to decompose its hidden state into 128 independent 2-offset conjunctions. The R^2 of ~0.83 reflects the fraction of variance these conjunctions capture; the remaining ~17% is higher-order interaction that the architecture cannot avoid.

3 Word-Length Encoding

Finding 2 (Word-length entanglement is 13× weaker)**.** *Subtracting the word-length direction from the hidden state costs only +0.54 bpc (enwik9) vs +7.3 bpc (1024B).*

	1024B	**enwik9**
Mean $\lvert r(h_j, \text{wl})\rvert$	0.014	0.0035
$r(h \cdot v_{\text{wl}}, \text{wl})$	0.58	0.47
Space reset $\lVert d \rVert$	5.31	7.10
Step-by-step subtract v_{wl}	+7.3	+0.54
Step-by-step subtract v_{tag}	—	+0.88
Step-by-step subtract both	—	+1.75
Write-in oracle wl	+0.51	+0.36
Post-hoc subtract wl	+0.15	+0.09
Random dir subtract (mean)	+2.3	+0.06

Table 2: Word-length encoding and intervention costs. The enwik9 model is far more robust to direction subtraction.

$\mathbf{W}_h$ eigenstructure. The top singular value of W_h is $\sigma_0 = 19.89$ (top neuron: h75 at -0.540), with $\lVert W_h v_{\text{pc1}} \rVert = 9.92$ and $\cos(W_h v_{\text{pc1}}, v_{\text{pc1}}) = 0.61$. For comparison, the 1024B model had $\lVert W_h v_{\text{wl}} \rVert = 2.48$ and $\cos = 0.79$. The enwik9 model has higher amplification (larger σ) but lower self-alignment: word-length information is propagated but scattered across dimensions rather than concentrated in a single eigenvector.

Robustness to random directions. Subtracting a random unit direction costs only +0.06 bpc (enwik9) vs +2.3 bpc (1024B). This confirms the shallow/distributed character: no single direction carries enough information to matter. The 1024B model concentrates information in specific directions, making it fragile to any perturbation; the enwik9 model distributes information, making it robust.

Residual word-length information. After subtracting v_{wl}, the rebuilt word-length direction has correlation $r = 0.37$ with actual word length (nonzero, confirming the distributed encoding). The maximum per-neuron correlation drops from 0.0103 to 0.23 after subtraction—word-length information is not removed, just rotated.

4 P6: Hebbian Weight Construction

Finding 3 (P6 is MIXED—overall r drops, large-weight r increases)**.** *The overall Hebbian correlation* $r(cov, W_h) = 0.38$ *(down from 0.56), but the large-weight correlation increases:* $r = 0.77$ *for* $|w| \geq 3.0$ *(up from 0.74).*

	1024B	**enwik9**
$r(\text{cov}, W_h)$ all	0.56	0.38
$r(\text{cov}, W_h)$ $|w| \geq 3$	0.74	**0.77**
Sign accuracy $|w| \geq 0.5$	72.7%	74.4%
Sign accuracy $|w| \geq 3.0$	—	93.3%
$r(\text{cov}, W_x)$ all	—	0.44
$r(\text{cov}, W_y)$ all	—	0.03
Hebbian all	7.44 bpc	7.44 bpc
Hebbian + opt W_y	1.15 bpc	1.90 bpc
Trained model	0.079 bpc	6.42 bpc

Table 3: Hebbian weight construction. The important weights (large $|w|$) are *better* predicted by Hebbian covariance on the enwik9 data, even though overall correlation drops.

The large-weight / small-weight split. Of the 16,384 entries in W_h, only 15 have $|w| \geq 3.0$ in the enwik9 model. These 15 entries—the structurally important connections—have $r = 0.77$ with Hebbian covariance and 93.3% sign accuracy (14/15 correct). The remaining entries are near-zero and dominated by noise, pulling the overall r down to 0.38.

Hebbian + optimized $\mathbf{W}_y$. Using Hebbian covariance for W_x, W_h, b_h and gradient-optimizing only W_y yields 1.90 bpc, far below the trained model's 6.42 bpc on this data. The Hebbian construction captures the recurrent dynamics well enough that a linear readout can decode them; the trained model cannot match this because it has moved on from these early bytes.

Interpolation. Blending 70% trained + 30% Hebbian W_h gives 7.13 bpc (worse than either alone), confirming that the Hebbian and trained weight matrices are structurally different even where they agree in sign.

5 P5: Fully Analytic Construction

Finding 4 (P5 is WRONG—analytic W_y below 5 bpc)**.** *The fully analytic construction achieves 4.21 bpc, well below the predicted threshold of 5.0. The loop is closed for a second model.*

Configuration	bpc
Uniform (baseline)	8.00
Analytic W_y (log-ratio, best scale)	**4.21**
Analytic W_y (PMI)	4.33
Naive Bayes (16 offsets, optimal temp)	4.40
Trained model (on this data)	6.42
Hebbian + optimized W_y	1.90

Table 4: Fully analytic construction. The analytic model outperforms the trained model on early data because the trained model has catastrophic forgetting.

The forgetting advantage. The analytic construction (4.21 bpc) outperforms the trained model (6.42 bpc) on the first 1024 bytes. This is not a flaw—it reveals that the analytic approach captures local data statistics that the online-trained model has overwritten. The analytic construction is inherently local and cannot suffer catastrophic forgetting.

6 Updated Prediction Scorecard

#	Prediction	Result	Verdict
P1	Margins decrease	Increase 6.5×	**WRONG**
P2	Distributed neurons	Top-1 = 10.4%	**RIGHT**
P3	Shallow offsets	78% at $d = 1$	**RIGHT**
P4	R^2 drops to 0.4–0.6	$R^2 = 0.830$	**WRONG**
P5	Analytic > 5 bpc	4.21 bpc	**WRONG**
P6	Hebbian r increases	Mixed (all: ↓, large: ↑)	**MIXED**
P7	Structural E→N→Q	Not tested	—

Table 5: Updated scorecard. Three wrong, two right, one mixed. The wrong predictions are wrong in the *interesting* direction: the architecture preserves more structure than expected.

7 Discussion

The central lesson: the 128-hidden tanh RNN imposes a specific computational structure regardless of training regime. The 2-offset conjunction factor map ($R^2 \approx 0.83$), the Boolean dynamics, and the Hebbian alignment of large weights are all architectural invariants.

What changes between models is the *character* of the invariant:

- The 1024B model is deep, concentrated, and fragile: a single neuron (h28) captures 99.7% of the bpc gap, offsets reach $d = 25$, and subtracting any direction is catastrophic.
- The enwik9 model is shallow, distributed, and robust: 60 neurons share the load, offsets peak at $d = 1$, and no single direction matters (+0.06 bpc for random subtraction).

Both are Boolean automata with $R^2 \approx 0.83$ factor maps. The architecture determines the *structure*; the training regime determines the *allocation* within that structure.