Scaling Byte-Level Kneser–Ney to 1.78 bpc on enwik9: Zero Structure, Just Counting

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Abstract

We scale interpolated Kneser–Ney smoothing on raw bytes from 10 MB to 1 GB (full enwik9), achieving **1.784 bpc** at order 6 with discount $D = 0.9$. No structure is imposed: no tokenization, no neural network, no event spaces—just exact n-gram counting with standard KN smoothing. The improvement rate is ∼0.07 bpc per doubling at small scale, slowing to ∼0.03 at large. Skip-offset models (d>6) and class-based output decompositions both fail to improve over sequential byte KN. This establishes a strong counting-only baseline for the Hutter Prize and quantifies how much of text compression is pure local redundancy versus structure.

1 Method

A single hash table stores all n-gram statistics up to order k. For each position t and each order $o \in \{1, \ldots, k\}$, we record:

- $c(w_{t-o}^{t-1}, w_t)$: continuation count (key: context + byte value),
- $c(w_{t-o}^{t-1}, \cdot)$: total count (key: context + sentinel 512),
- $\tau(w_{t-o}^{t-1})$: type count (key: context + sentinel 513), incremented only when a new byte type appears in context.

Interpolated KN. The prediction builds bottom-up from the marginal:

$$P(w_t \mid w_{t-o}^{t-1}) = \frac{\max(c - D, 0)}{c(\cdot)} + \frac{D \cdot \tau}{c(\cdot)} \cdot P(w_t \mid w_{t-o+1}^{t-1}),$$

recursing down to the unigram marginal $P(w_t)$. All orders contribute, unlike standard (backoff) KN which stops at the highest matching order. Interpolated KN consistently outperforms backoff by 0.005–0.01 bpc.

Hash table. Open-addressing with FNV-1a hashing, 12 bytes per entry (8-byte key + 4-byte count). 128M entries = 1.5 GB. At full enwik9 (800M train), order 6 fills the table to 99.9%. A 256M-entry table (3 GB) would avoid saturation but requires >5 GB total—above our 7.8 GB memory budget.

Data	KN-5i best	KN-6i best	HT fill
10M	2.286	—	5%
20M	2.175	—	8%
50M	2.093	2.093	24%
100M	2.038	2.001	32%
200M	1.984	1.927	48%
400M	1.960	1.889	73%
800M	1.945	1.859	97%
1B	1.860	**1.784**	100%

Table 1: Best interpolated KN bpc (bits per character) on held-out test data. All use discount $D = 0.8$ except 1B KN-6i which uses $D = 0.9$. HT fill is for KN-6 at the 128M-entry table.

2 Results

Diminishing returns. Per-doubling improvement for KN-6i: 0.074 (100M→200M), 0.038 (200M→400M), 0.030 (400M→800M). The rate is halving per doubling—logarithmic convergence. Extrapolating, 10B data (if available) would give ∼1.7 bpc.

Order saturation. At 100M, order 7 is worse than order 6 (2.008 vs 2.001)—overfitting. At 200M, order 7 is slightly better (1.918 vs 1.927) but requires 92% HT. At 400M+, order 7 overflows the HT completely (100% fill at 234M/320M train). The optimal order grows sublogarithmically with data size.

3 Negative Results

Skip offsets add nothing. We built separate KN models on non-sequential offsets (d=7,8,9,10,11,12), selected by mutual information. At 20M: skip KN-4 alone = 4.65 bpc. Product-of-experts combination: best $\alpha = 1.0$ (byte KN alone wins). The sequential offsets d=1..6 completely dominate at byte level.

Class-based output decomposition fails. Decomposing $P(w \mid \text{ctx}) = P(\text{class} \mid \text{event ctx}) \cdot P(w \mid \text{class}, \text{byte ctx})$ with frequency-based output classes (K=4,8,16,32) never beats byte KN at any scale from 65K to 10M. The Zipfian byte distribution makes classes extremely imbalanced.

4 Implications for the Universal Model

What 1.784 bpc represents. This is the amount of enwik9 redundancy capturable by exact local pattern matching up to 6 bytes. No inter-word, no syntactic, no semantic structure is used. The remaining ∼1.4 bpc (from the ∼0.4 bpc entropy floor) is "structure" in the CMP sense—it requires event spaces, not just byte counting.

Comparison to other methods.

- sat-rnn (128 hidden, 110M train): 2.81 bpc at byte level (0.079 bpc at 100K with custom eval—different regime)

- Byte KN-6i at 200M: 1.927 bpc (already beats sat-rnn)
- LSTM-based compressors: ∼1.3 bpc
- Transformers (GPT-scale): ∼0.9 bpc

The KN baseline at 1.784 is remarkable—it captures ∼78% of the known-compressible redundancy (∼8 - 0.4 = $7.6bpcrange)usingonly6-gramstatistics.$

The counting floor. As data $\to \infty$, byte KN converges to the k-th order entropy. For English at order 6, this appears to be ∼1.7 bpc. The gap between this floor and transformer-level compression (∼0.9 bpc) is the "structural redundancy"—information that requires understanding words, syntax, and semantics, not just byte patterns.

HT saturation as the real bottleneck. At 1B, the 128M-entry HT is 99.9% full. Many 6-grams are being lost to hash collisions. A 256M-entry table (3 GB HT + 1 GB data) was OOM-killed on our 7.8 GB machine. With adequate memory, unsaturated order 6 would almost certainly push below 1.75 bpc; order 7 might reach ∼1.7 bpc. The algorithm is memory-limited, not data-limited.

5 Conclusion

Byte-level KN smoothing scales cleanly to full enwik9: 1.784 bpc with no structure beyond 6-gram counting. This establishes both a practical baseline and a theoretical reference point. Everything below 1.784 requires the Universal Model's event spaces, pattern chains, or learned representations. The counting foundation is now solid; the structural frontier begins here.