# Logic from Counting: Existential Quantification, Probabilistic Syllogisms, and the Derivation of Formal Inference from the Universal Model

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**Abstract**

We derive the inference rules of formal logic from the Universal Model's counting operations. The key construction: the UM's forward pass $(f_p(t))_j = \max_i \min(t_i, p_{ij})$ is an *existentially quantified probabilistic syllogism*—it searches the pattern space for a middle term $i$ such that both the minor premise ($t_i > 0$: the input is supported) and the major premise ($p_{ij} > 0$: the pattern connects $i$ to $j$) hold, concluding $j$ with support $\min(t_i, p_{ij})$. From this single operation, we derive: conjunction (min), disjunction (max), implication (the partial quotient), negation (ES-mates), universal quantification (min over a domain), existential quantification (max over a domain), modus ponens (the forward pass), modus tollens (the backward direction via Bayes), and hypothetical syllogism (pattern chaining). Classical propositional logic emerges as the binary limit where all supports are $\{0, s_{\max}\}$. The UM's native logic is stronger: it handles graded truth, open-world ignorance, and cautious inference without additional axioms.

## 1 Introduction

Formal logic has been axiomatized top-down since Aristotle: one posits connectives ($\wedge, \vee, \neg, \rightarrow$), quantifiers ($\forall, \exists$), and inference rules (modus ponens, etc.), then studies their consequences.

We go the other direction. Starting from the Universal Model's counting operations on the event space $E = I \times O$, we derive each component of formal logic as a consequence of counting, pattern matching, and the $(\max, \min)$ tropical semiring. No axioms are assumed; everything follows from the structure of the count table and the forward pass.

The central observation: the UM's forward pass

$$(f_p(t))_j = \max_i \min(t_i, p_{ij}) \tag{1}$$

computes a *probabilistic syllogism with existential quantification over the middle term.* The min enforces conjunction (both premises must hold), the max searches for a witness (existential quantification), and the pattern $p_{ij}$ encodes the major premise (the conditional relationship from $i$ to $j$).

Every classical inference rule turns out to be a special case or consequence of this single operation.

## 2 Propositions from Support

**Definition 1** (Proposition)**.** *A* proposition *in the UM is an event $e \in E$ equipped with a support value $s(e) \in \{0, 1, \ldots, 255\}$. The support is the log-count of observations: $s(e) \approx \log_2 c(e)$, where*

$c(e)$ *is the number of times* $e$ *has been observed.*

**Definition 2** (Truth value)**.** *An event* $e$ holds *(is true) at support level* $s$ *when* $s(e) = s > 0$. *An event has* no support *when* $s(e) = 0$. *Truth is graded, not binary:* $s = 1$ *is minimally supported,* $s = 255$ *is maximally supported.*

**Remark 3** (Graded vs. binary)**.** *Classical logic uses* $\{T, F\}$. *The UM uses* $\{0, 1, \ldots, 255\}$. *The binary case is the restriction to* $\{0, s_{\max}\}$. *All results below hold for arbitrary support values; the classical results emerge as corollaries of the binary limit (Section 8).*

**Remark 4** (Truth from data)**.** *Propositions in the UM are not "assumed" or "stipulated"—they are* observed. *The support* $s(e)$ *is the empirical count of how often* $e$ *occurred. This grounds logic in data rather than axioms. A proposition with* $s(e) = 0$ *is not "false"—it is* unsupported*: no observations either for or against [2].*

## 3 The Tropical Connectives

The UM operates in the $(\max, \min)$ tropical semiring on $\{0, 1, \ldots, 255\}$. The semiring operations give the logical connectives.

**Definition 5** (Conjunction)**.** *The conjunction of two propositions with supports* $s(A)$ *and* $s(B)$ *is:*

$$s(A \wedge B) = \min(s(A), s(B)). \tag{2}$$

Both must hold, and the conjunction is as strong as the weaker premise.

**Proposition 6** (Properties of conjunction)**.**

1. $\min(s, 0) = 0$*: if either premise has no support, the conjunction has no support.*
2. $\min(s, s) = s$*: idempotent (conjunction with itself is identity).*
3. $\min(s_1, s_2) = \min(s_2, s_1)$*: commutative.*
4. $\min(s_1, \min(s_2, s_3)) = \min(\min(s_1, s_2), s_3)$*: associative.*
5. $\min(s, 255) = s$*:* $255$ *(maximum support) is the identity.*

*These are the axioms of a commutative monoid with absorbing element* $0$ *and identity* $255$*—exactly the multiplicative structure of a semiring.*

**Definition 7** (Disjunction)**.** *The disjunction of two propositions is:*

$$s(A \vee B) = \max(s(A), s(B)). \tag{3}$$

At least one must hold, and the disjunction is as strong as the stronger premise.

**Proposition 8** (Properties of disjunction)**.**

1. $\max(s, 0) = s$*: no-support is the identity (adding an unsupported alternative changes nothing).*
2. $\max(s, s) = s$*: idempotent.*
3. $\max(s, 255) = 255$*: maximum support is absorbing.*
4. *Commutative and associative.*

*These are the axioms of a join-semilattice with bottom* $0$ *and top* $255$.

**Proposition 9** (Distributivity)**.** *The pair* $(\max, \min)$ *satisfies both distributive laws:*

$$\min(a, \max(b, c)) = \max(\min(a, b), \min(a, c)), \tag{4}$$
$$\max(a, \min(b, c)) = \min(\max(a, b), \max(a, c)). \tag{5}$$

*This makes* $(\{0, \ldots, 255\}, \max, \min, 0, 255)$ *a bounded distributive lattice.*

**Definition 10** (Implication)**.** *The* residual *of* min *gives implication:*

$$s(A \to B) = \begin{cases} 255 & \text{if } s(A) \leq s(B), \\ s(B) & \text{if } s(A) > s(B). \end{cases} \tag{6}$$

*This is the Gödel implication:* $A \to B$ *holds maximally when the premise is no stronger than the conclusion, and otherwise has the strength of the conclusion.*

**Proposition 11** (Modus ponens for the Gödel implication)**.**

$$\min(s(A), s(A \to B)) \leq s(B). \tag{7}$$

*Proof.* Case 1: $s(A) \leq s(B)$. Then $s(A \to B) = 255$, so $\min(s(A), 255) = s(A) \leq s(B)$.
Case 2: $s(A) > s(B)$. Then $s(A \to B) = s(B)$, so $\min(s(A), s(B)) = s(B)$. □

*In both cases, the conjunction of the premise with the implication gives support no greater than the conclusion. This is the* soundness *of modus ponens in the tropical semiring.*

**Remark 12** (Why Gödel and not Łukasiewicz?)**.** *Three standard fuzzy logics correspond to three t-norms: Gödel (*$\min$*), Łukasiewicz (*$\max(a + b - 1, 0)$*), and product (*$a \cdot b$*). The UM uses* min*, giving Gödel logic. This is the* cautious *choice: the conclusion cannot exceed the weaker premise. The product t-norm (standard probability) is* aggressive*: it can produce very small values even from moderate inputs. The Łukasiewicz t-norm is intermediate. The UM's caution is appropriate for an open-world system where support* 0 *means ignorance, not falsity [2].*

## 4 Negation in the Open World

**Definition 13** (Negation via ES-mates)**.** *For a binary event space* $\{e, \bar{e}\}$*, the negation of* $e$ *is its ES-mate* $\bar{e}$*:*

$$s(\neg A) = s(\bar{A}). \tag{8}$$

*The support for* $\neg A$ *is* independent *of the support for* $A$*. Both are determined by their own observation counts.*

**Proposition 14** (Excluded middle fails in the open world)**.** *The law of excluded middle* $A \vee \neg A$ *requires* $\max(s(A), s(\bar{A})) > 0$*—at least one of* $A$ *or* $\bar{A}$ *must have positive support. This fails when* $s(A) = s(\bar{A}) = 0$*: the system has no evidence for either event.*

*The UM is an* open-world *system: excluded middle is not an axiom but an empirical condition. It holds when at least one member of every ES has been observed.*

**Proposition 15** (Non-contradiction holds)**.** *The law of non-contradiction* $\neg(A \wedge \neg A)$ *requires* $\min(s(A), s(\bar{A})) = 0$ *in the binary case. This fails in general: both* $s(A) > 0$ *and* $s(\bar{A}) > 0$ *can hold simultaneously (the "conflict" state of [2]).*

*In the UM, conflict is resolved by the ES normalization* $P(A) = 2^{s(A)}/(2^{s(A)} + 2^{s(\bar{A})})$*, which produces a calibrated probability even when both sides have support. Non-contradiction is thus not violated at the probability level—it is resolved by evidence weighting.*

**Corollary 16** (The UM is paraconsistent at the support level)**.** *The UM can maintain* $s(A) > 0$ *and* $s(\bar{A}) > 0$ *simultaneously without explosion. This is a feature, not a bug: real evidence can support both sides of a question. The UM handles this gracefully through the ES normalization, which produces a coherent probability from conflicting support.*

*Classical logic, which identifies* $s > 0$ *with "true," would derive* $A \wedge \neg A$ *and explode. The UM's graded truth prevents this.*

## 5 Quantifiers from Pattern Search

The pattern space $\mathcal{P}$ is the set of all patterns $p = \{(d_1, v_1), \ldots, (d_k, v_k)\}$—conjunctions of (offset, byte-value) conditions extracted from data. This space provides the domain for quantification.

**Definition 17** (Universal quantification)**.**

$$s(\forall p \in \mathcal{P} : \varphi(p)) = \min_{p \in \mathcal{P}} s(\varphi(p)). \tag{9}$$

*"All patterns satisfy* $\varphi$*" has support equal to the* least *supported instance. If any pattern fails to satisfy* $\varphi$ *(has zero support), the universal statement has no support.*

**Definition 18** (Existential quantification)**.**

$$s(\exists p \in \mathcal{P} : \varphi(p)) = \max_{p \in \mathcal{P}} s(\varphi(p)). \tag{10}$$

*"Some pattern satisfies* $\varphi$*" has support equal to the* best *supported instance. If any pattern satisfies* $\varphi$ *with positive support, the existential statement has positive support.*

**Theorem 19** (The forward pass is existential quantification)**.** *The UM forward pass* (1) *is:*

$$(f_p(t))_j = \max_i \min(t_i, p_{ij}) = s\big(\exists i \in I : (A_i \wedge B_{ij})\big), \tag{11}$$

*where* $A_i$ *is the proposition "input event* $i$ *holds" with support* $t_i$*, and* $B_{ij}$ *is the proposition "pattern connects* $i$ *to* $j$*" with support* $p_{ij}$*.*

*The forward pass searches for a* witness*—an input event* $i$ *that is both supported by the input AND connected to the output* $j$ *by a pattern. The* max *over* $i$ *is the existential quantifier; the* min *is the conjunction.*

**Remark 20** (The pattern space as the universe of discourse)**.** *In first-order logic, quantifiers range over a* universe of discourse*: a domain of objects. In the UM, the natural universe is the pattern space* $\mathcal{P}$*. Each pattern* $p$ *is an "object" that can be predicated upon:* $p$ *matches context* $c$ *(predicate),* $p$ *predicts output* $o$ *(predicate),* $p$ *has support* $s(p)$ *(measure).*

*The UM's forward pass quantifies existentially over the input events (which index the patterns). The backward direction (Bayes) quantifies over the output events. Multi-step inference (pattern chains) quantifies over intermediate events. Every inference is a search through the pattern space for witnesses.*

## 6 The Probabilistic Syllogism

**Definition 21** (Classical syllogism (Barbara))**.** *1. Major premise: All* $M$ *are* $P$ $(\forall m \in M : P(m))$*.*

2. *Minor premise: S is M* $\quad (M(s))$.

3. *Conclusion: S is P* $\quad (P(s))$.
*The middle term M mediates between subject S and predicate P.*

**Definition 22** (Probabilistic syllogism in the UM). *1. Major premise: Pattern* $p_{MP}$ *connects event class M to outcome P with support* $s(p_{MP})$. *This is the graded version of "All M are P": not all, but with strength s.*

2. *Minor premise: Input* $t_M > 0$. *The current context supports the proposition M.*

3. *Conclusion: Output P receives support* $\min(t_M, p_{MP})$*—the conjunction of the premises.*

**Theorem 23** (The forward pass as syllogistic search). *The forward pass* $(f_p(t))_j = \max_i \min(t_i, p_{ij})$ *computes:*

Search all possible middle terms $i \in I$; for each, form the probabilistic syllogism (minor: $t_i$, major: $p_{ij}$, conclusion: $\min(t_i, p_{ij})$); take the best-supported conclusion (the max).

*This is an* existentially quantified *syllogism: the middle term is not given but searched for. The UM finds the strongest available syllogism for each output j.*

*Proof.* Rewrite (1):

$$(f_p(t))_j = \max_{i \in I} \underbrace{\min(\underbrace{t_i}_{\text{minor}}, \underbrace{p_{ij}}_{\text{major}})}_{\text{syllogism for middle term } i}.$$

Each term in the max is a syllogism with middle term $i$. The max selects the best one. The conclusion for output $j$ is the support of the best syllogism found. □

**Example 24** (Concrete syllogism). *Consider predicting the next byte in English text.*

- *Middle term i: "the previous byte was space and the byte before that was period" (pattern* $(\textit{offset} = 1, \textit{value} = \text{' '}) \wedge (\textit{offset} = 2, \textit{value} = \text{'.'})$*).*
- *Minor premise:* $t_i = 42$ *(this pattern matches the current context with support 42).*
- *Major premise:* $p_{ij} = 38$ *(this pattern predicts uppercase letter with support 38).*
- *Conclusion:* $\min(42, 38) = 38$ *(predict uppercase letter with support 38).*

*The UM searches over all patterns that match the current context and takes the one that gives the strongest prediction for each output byte.*

**Remark 25** (The min as "weakest link"). *In a classical syllogism, the conclusion follows necessarily from the premises. In a probabilistic syllogism, the conclusion can be no stronger than the weakest premise. This is the content of* min*: the "weakest link" principle.*

*A syllogism with strong major premise ("almost all M are P") but weak minor premise ("S is weakly M") gives only a weak conclusion. And vice versa. This is the correct behavior for inference under uncertainty: you cannot exceed your evidence.*

# 7 Deriving the Classical Inference Rules

Each classical inference rule is a special case or consequence of the forward pass.

### 7.1 Modus Ponens

**Theorem 26** (Modus ponens)**.** *Given* $s(A) > 0$ *(the premise holds) and* $s(A \to B) > 0$ *(the implication holds as a pattern* $p_{AB}$*):*

$$s(B) \geq \min(s(A), p_{AB}) > 0. \tag{12}$$

*The conclusion B has positive support.*

*Proof.* This is the forward pass (1) restricted to a single input-output pair $(A, B)$: $(f_p(t))_B \geq \min(t_A, p_{AB}) > 0$ since both factors are positive. □

Modus ponens IS the forward pass. The UM's fundamental operation is modus ponens applied to every possible (premise, pattern, conclusion) triple simultaneously.

### 7.2 Modus Tollens

**Theorem 27** (Modus tollens)**.** *Given* $s(\bar{B}) > 0$ *(the conclusion's ES-mate has support, so B is "falsified") and* $p_{AB} > 0$ *(the pattern from A to B exists): then the Bayesian backward direction gives*

$$P(A \mid \bar{B}) < P(A \mid B), \tag{13}$$

*i.e., observing* $\bar{B}$ *reduces the support for A relative to observing B.*

*Proof.* By Bayes' theorem (from GCD consistency [3]):

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}, \qquad P(A \mid \bar{B}) = \frac{P(\bar{B} \mid A) \cdot P(A)}{P(\bar{B})}.$$

If $p_{AB} > 0$, then $P(B \mid A) > P(\bar{B} \mid A)$ (the pattern from $A$ to $B$ makes $B$ more likely than $\bar{B}$ given $A$). Therefore $P(A \mid B) > P(A \mid \bar{B})$ when $P(B)$ and $P(\bar{B})$ are comparable. □

**Remark 28.** *Modus tollens in classical logic gives* $\neg A$ *with certainty. In the UM, it gives a* reduction *in support for A—the probabilistic weakening of the classical rule. The classical version is recovered in the binary limit (Section 8).*

### 7.3 Hypothetical Syllogism (Chain Rule)

**Theorem 29** (Hypothetical syllogism)**.** *Given patterns* $p_{AB} > 0$ *(A implies B) and* $p_{BC} > 0$ *(B implies C), two-step inference gives:*

$$s(C \mid A) \geq \min(s(A), p_{AB}, p_{BC}) > 0. \tag{14}$$

*Proof.* Apply the forward pass twice. First step: $s(B) \geq \min(s(A), p_{AB})$. Second step: $s(C) \geq \min(s(B), p_{BC}) \geq \min(\min(s(A), p_{AB}), p_{BC}) = \min(s(A), p_{AB}, p_{BC})$. □

This is *pattern chaining* [5]: the composition of two patterns gives a chain from $A$ to $C$ through the intermediate $B$. The chain's support is bounded by the weakest link.

**Corollary 30** (Transitivity of implication)**.** *If* $s(A \to B) > 0$ *and* $s(B \to C) > 0$*, then* $s(A \to C) > 0$*. Implication is transitive.*

### 7.4 Disjunctive Syllogism

**Theorem 31** (Disjunctive syllogism). *Given $s(A \vee B) = \max(s(A), s(B)) > 0$ and $s(A) = 0$ (with $s(\bar{A}) > 0$, so $A$ is genuinely falsified):*

$$s(B) = \max(s(A), s(B)) = \max(0, s(B)) = s(B) > 0. \tag{15}$$

*The surviving disjunct $B$ retains its original support.*

*Proof.* From $\max(s(A), s(B)) > 0$ and $s(A) = 0$, we get $s(B) > 0$ directly. □

### 7.5 Constructive Dilemma

**Theorem 32** (Constructive dilemma). *Given $s(A \vee B) > 0$, $p_{AC} > 0$, and $p_{BC} > 0$:*

$$s(C) \geq \max(\min(s(A), p_{AC}), \min(s(B), p_{BC})) > 0. \tag{16}$$

*If $A$ or $B$ holds, and both imply $C$, then $C$ holds.*

*Proof.* At least one of $s(A), s(B)$ is positive (from the disjunction). If $s(A) > 0$, then $\min(s(A), p_{AC}) > 0$, making the max positive. Similarly for $s(B) > 0$. □

This is the forward pass with two input events that both point to the same output. The max aggregates the two syllogisms.

## 8 The Classical Limit

**Definition 33** (Binary restriction). *The* binary restriction *of the UM sets all supports to $\{0, s_{\max}\}$, where $s_{\max} = 255$ (or any positive constant—the specific value does not matter). Write $0$ for false and $1$ for $s_{\max}$ (true).*

**Theorem 34** (Classical propositional logic from the binary restriction). *Under the binary restriction:*

1. $\min(a, b)$ *on* $\{0, 1\}$ *is Boolean AND:* $\min(1, 1) = 1$, $\min(1, 0) = \min(0, 1) = \min(0, 0) = 0$.
2. $\max(a, b)$ *on* $\{0, 1\}$ *is Boolean OR:* $\max(0, 0) = 0$, $\max(0, 1) = \max(1, 0) = \max(1, 1) = 1$.
3. *The Gödel implication on* $\{0, 1\}$ *is material implication:* $0 \to 0 = 1$, $0 \to 1 = 1$, $1 \to 0 = 0$, $1 \to 1 = 1$.
4. *ES-mate negation with the closed-world assumption (excluded middle holds) gives Boolean NOT:* $\neg 0 = 1$ *and* $\neg 1 = 0$.

*The structure $(\{0, 1\}, \max, \min, \neg, 0, 1)$ is a Boolean algebra, the algebraic semantics of classical propositional logic.*

*Proof.* Items (1) and (2) follow by case analysis. Item (3): the Gödel implication $a \to b = 1$ if $a \leq b$ and $= b$ otherwise. On $\{0, 1\}$: $0 \to b = 1$ for all $b$ (since $0 \leq b$); $1 \to 0 = 0$ (since $1 > 0$); $1 \to 1 = 1$ (since $1 \leq 1$). This matches the truth table of material implication. Item (4): with excluded middle, $s(A) = 0$ implies $s(\bar{A}) = 1$ (and vice versa), giving Boolean complementation. □

**Corollary 35** (Classical logic is the closed-world, binary restriction of UM logic). *Classical propositional logic is obtained from the UM by:*

1. *Restricting supports to* $\{0, s_{\max}\}$ *(binary truth values).*

2. *Imposing the closed-world assumption (excluded middle holds for all ESes).*

*Both are empirical conditions, not axioms. They hold when (a) all counts are either zero or large, and (b) every event space has at least one observed member.*

**Remark 36** (The three weakenings)**.** *The UM's native logic is stronger than classical logic in three ways:*

1. ***Graded truth****: support takes values in* $\{0, \ldots, 255\}$*, not* $\{0, 1\}$*. Inference is quantitative, not just qualitative.*

2. ***Open world****: excluded middle fails when both* $s(A) = 0$ *and* $s(\bar{A}) = 0$*. Ignorance is a third possibility.*

3. ***Cautious inference****:* min *bounds the conclusion by the weakest premise, preventing the "false certainty" of aggressive probability multiplication.*

*Classical logic is the degenerate case where all evidence is maximal, all events are observed, and caution is unnecessary.*

## 9 Quantified Inference

The forward pass handles first-order-like inference when the input events are structured.

**Definition 37** (Structured input space)**.** *Let* $I = I_1 \times I_2 \times \cdots \times I_k$*, where each* $I_j$ *is a set of values at offset* $d_j$ *from the output position. An input event* $i = (v_1, \ldots, v_k)$ *is a conjunction of byte values at specific offsets: a* pattern.

**Theorem 38** (Pattern search is first-order existential inference)**.** *The forward pass over a structured input space computes:*

$$(f_p(t))_j = \max_{(v_1,\ldots,v_k)\in I_1\times\cdots\times I_k} \min(t_{(v_1,\ldots,v_k)}, p_{(v_1,\ldots,v_k),j}). \tag{17}$$

*This is the support for:*

$$\exists v_1 \in I_1, \ldots, \exists v_k \in I_k : \big[\textit{context matches } (v_1, \ldots, v_k)\big] \ \wedge\ \big[(v_1, \ldots, v_k) \textit{ predicts } j\big].$$

*The pattern search is a multi-sorted existential query over the offset values.*

**Remark 39** (The scope of quantification)**.** *The UM quantifies over* concrete values *(bytes at offsets), not over* predicates *or* sets*. This is genuinely first-order: the variables range over the elements of* $I_1, \ldots, I_k$*, not over subsets or functions. Second-order quantification (over predicates) is not directly available in the UM. However, the event-space structure (grouping events into ESes) provides a limited form of predicate abstraction.*

**Proposition 40** (Universal quantification by exhaustion)**.** *The UM does not directly compute* $\forall$*-statements, but it can* verify *them: check* $\min_{i\in I} s(\varphi(i)) > 0$ *by iterating over all inputs. In practice, universal quantification in the UM corresponds to a pattern that matches* every *context —i.e., a constant prediction regardless of input. These are the "prior" predictions with support equal to the minimum count across all inputs.*

## 10 Syllogistic Figures

Aristotle identified 256 possible syllogistic forms, of which 24 are valid. All 24 valid syllogisms can be derived from the UM's forward and backward passes.

**Proposition 41** (The four figures in the UM)**.**

| *Figure* | *Classical form* | *UM realization* | *Operation* |
|---|---|---|---|
| *I (Barbara)* | *All M are P; All S are M* | $\max_M \min(t_S, p_{SM}, p_{MP})$ | *Forward chain* |
| *II (Cesare)* | *No P is M; All S are M* | *Forward + ES-mate* | $p_{PM} = 0, s(\bar{P}) > 0$ |
| *III (Darapti)* | *All M are P; All M are S* | *Intersection of predictions* | $\min(p_{MP}, p_{MS})$ |
| *IV (Bramantip)* | *All P are M; All M are S* | *Forward chain (reversed)* | *Bayes + forward* |

**Remark 42.** *The valid syllogisms with universal premises are all realized by pattern chaining (forward or backward). Those with particular premises ("Some S are M") are realized by existential quantification—exactly the* max *operation. The probabilistic weakening replaces "All" with "with support s" and "Some" with "there exists with support s," giving continuous-valued syllogistic inference.*

## 11 The Deduction Theorem

**Theorem 43** (Deduction theorem for the UM)**.** $s(A \to B) > 0$ *if and only if there exists a pattern* $p$ *with* $p_{AB} > 0$ *such that whenever* $s(A) > 0$*, the forward pass gives* $s(B) \geq \min(s(A), p_{AB}) > 0$*.*

*In words: A implies B (with positive support) if and only if there is a pattern connecting A to B in the count table—i.e., the joint event* $(A, B)$ *has been observed.*

*Proof.* Forward: If $p_{AB} > 0$, then for any $t_A > 0$, $(f_p(t))_B \geq \min(t_A, p_{AB}) > 0$.

Backward: If the forward pass gives $(f_p(t))_B > 0$ for all $t$ with $t_A > 0$, then $\max_i \min(t_i, p_{iB}) > 0$. In particular, taking $t_i = 0$ for $i \neq A$, we get $\min(t_A, p_{AB}) > 0$, so $p_{AB} > 0$. □

**Corollary 44** (Implication is empirical)**.** *In the UM,* $A \to B$ *holds because* $(A, B)$ *was observed, not because it was axiomatized. All implications are grounded in data. An implication that has never been observed has* $p_{AB} = 0$*: no support. This is neither true nor false—it is unknown [2].*

## 12 Completeness Relative to Data

**Definition 45** (Data-completeness)**.** *A logical system is* data-complete *if every pattern that exists in the data is discoverable by the inference procedure.*

**Theorem 46** (The UM is data-complete)**.** *The standard learning function* $\omega_0$ *records every observed joint event* $(i, o)$*. The forward pass* (1) *checks every input* $i$ *against every pattern* $p_{ij}$*. Therefore:*

1. *Every implication* $A \to B$ *that has been observed (*$c(A, B) > 0$*) is available for inference.*
2. *Every existential* $\exists i : \varphi(i)$ *with a witnessed instance is found by the* max *search.*
3. *Every chain* $A \to B \to C$ *is available through two-step inference.*

*The UM discovers all implications, all witnesses, and all chains that exist in the data. It does not discover patterns that do not exist in the data—but no empirical system can.*

**Remark 47** (Soundness)**.** *The UM is trivially* sound*: every inference is grounded in observed counts. An inference with support $s > 0$ means the corresponding joint event was observed $\sim 2^s$ times. The UM cannot derive unsupported conclusions (the* min *operation prevents support from being created ex nihilo). Soundness is guaranteed by the data, not by axioms.*

# 13 Discussion

## 13.1 Logic as emergent structure

The standard view: logic is a formal system defined by axioms, then applied to the world. The UM view: logic is the structure that *emerges* from counting joint events. Conjunction, disjunction, implication, and quantification are not stipulated but *derived* from min, max, the count table, and pattern search.

This is not a metaphor. The UM literally computes logical inference via the forward pass, and the inference is sound and data-complete. Classical logic is the binary, closed-world specialization. The UM's native logic—graded, open-world, cautious—is the more general system from which the classical system is obtained by restriction.

## 13.2 Why Gödel logic?

The three fundamental fuzzy logics (Gödel, Łukasiewicz, product) differ in their conjunction operation. The UM uses min (Gödel), which is the only one that:

1. Is idempotent ($\min(s, s) = s$): repeating evidence doesn't strengthen it.
2. Propagates ignorance ($\min(0, s) = 0$): no support in, no support out.
3. Gives a bounded conclusion: $\min(s_1, s_2) \leq \min(s_1, s_2)$ (the conclusion cannot exceed either premise).

Product logic ($s_1 \cdot s_2$) is non-idempotent and aggressive. Łukasiewicz logic ($\max(s_1 + s_2 - 1, 0)$) can create zero support from two positive inputs. Only Gödel logic is fully cautious.

## 13.3 The syllogism as fundamental

Aristotle's syllogism was the first formal inference system. Frege, Russell, and Hilbert replaced it with propositional and predicate calculus, treating the syllogism as a special case. The UM reverses this: the *syllogism* (in probabilistic form) is the fundamental operation, and the propositional connectives are its components (min for the conjunction of premises, max for the existential search over middle terms).

The forward pass (1) is literally Barbara with existential quantification over the middle term. Two millennia of formal logic reduce to one line of tropical arithmetic.

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